

## Course Outline

School Name: **UMC High School**

Department Name: **MATHEMATICS**

**Ministry of Education Course Title: Calculus and Vectors, Gr 12, University Preparation**

**Grade Level: 12, University Preparation**

Ministry Course Code: **MCV4UX**

Instructor:

Developed by: Karthika Manickavasakar

Date: July 2013

Revision Date: Sept 2016

Developed from: Mathematics, the Ontario Curriculum, Grades 11 and 12, 2007

Required Texts:

Textbook: *Calculus and Vectors 12*, Nelson (2008)

Supplementary texts (Supplied by the instructor):

- Teacher-made Worksheets
- Graphing Calculators & Computers

**Prerequisite:** The new Advanced Functions course (MHF4U) must be taken prior to or concurrently with Calculus and Vectors (MCV4U).

Credits: 1

Length: 110 hours

Principal: \_\_\_\_\_

Head Teacher: \_\_\_\_\_

## Course Description

This course builds on students' previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modelling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

## Overall Curriculum Expectations

By the end of this course, students will:

### **Strand A: *Rates of Change***

1. demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit;
2. graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative;
3. verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems.

### **Strand B: *Derivatives and Their Applications***

4. make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;
5. solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.

### **Strand C: *Geometry and Algebra of Vectors***

6. demonstrate an understanding of vectors in two-space and three-space by representing them algebraically and geometrically and by recognizing their applications;
7. perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications;
8. distinguish between the geometric representations of a single linear equation or a system of two linear equations in two-space and three-space, and determine different geometric configurations of lines and planes in three-space;
9. represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections

## Course Content

<b>Chapter 1</b>	<b>Introduction to Calculus</b> Strand: A Overall Expectations: 1	14 hours
<b>Chapter 2</b>	<b>Derivatives</b> Strand: A Overall Expectations: 2-3	11 hours
<b>Chapter 3</b>	<b>Derivatives and their Applications</b> Strand: B Overall Expectations: 4-5	11 hours
<b>Chapter 4</b>	<b>Curve Sketching</b> Strand: B Overall Expectations: 4-5	11 hours
<b>Chapter 5</b>	<b>Derivatives of Exponential and Trigonometric Functions</b> Strand: A Overall Expectations: 2-3	14 hours
<b>Chapter 6</b>	<b>An Introduction to Vectors</b> Strand: C Overall Expectations: 6-7	14 hours
<b>Chapter 7</b>	<b>Applications of Vectors</b> Strand: C Overall Expectations: 7	14 hours
<b>Chapter 8</b>	<b>Equations of Lines and Planes</b> Strand: C Overall Expectations: 8-9	10 hours
<b>Chapter 9</b>	<b>Relationships Between Points, Lines, and Planes</b> Strand: C Overall Expectations: 8-9	9 hours
	<b>Final Exam/Summative Evaluation</b> <b>Strands:</b> A. <i>Rate change</i> B. <i>Derivatives and their applications</i> C. <i>Geometry and algebra of vectors</i> <b>Overall Expectations:</b> A1-A3, B1-B2,C1-C4	2 hours

TOTAL 110hours

## Chapter Descriptions

### Chapter 1 – Introduction to Calculus

Time: 14 hours

#### Description

In this chapter, the students will review the concepts of average and instantaneous rate of change to determine the rate of change over an interval or at a single point. Students are introduced to the concept of limit which involves getting close to a value but never getting to that value. They solve limit using variety of method like factoring, rationalization and change of variables. Students determine if the function is continuous or not by using the concepts of one sided limits.

#### Specific Expectations

*A. Rate of Change: 1.1, 1.2, 1.3, 1.4, 1.5,1.6*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 1 Test	x	x	x	x
Chapter 1 Assignment	x	x	x	x

## Chapter 2 – Derivatives

Time: 11 hours

### Description

Students use the first principle definition of derivative to determine the derivative of simple polynomial functions. They use the power rule, the product rule, the quotient rule and the chain rule to determine the derivative of more complex polynomial and rational functions.

### Specific Expectations

A. *Rate of change: 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 3.4, 3.5*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 2 Test	x	x	x	x

## Chapter 3 – Derivatives and their Applications

Time: 11 hours

### Description

In this chapter, student use derivatives to solve optimization problems and problems involving position, velocity and acceleration. Students use the first derivative to determine the interval of increase and decrease of a function. They use that information to identify if a function has a local maximum or minimum. Students also use the concept of limit to determine the horizontal asymptote of a rational function. Student use the second derivative to identify the interval of concavity and use the second derivative test to determine if a function has a local maximum or minimum at the critical points. In the end of the unit, students are able to use the curve sketching algorithm ( x-intercepts, y-intercepts, critical points, interval of increase and decrease, point of infection, and interval of concavity) to sketch a proper graph of a polynomial and rational functions.

### Specific Expectations

*B. Derivatives and their Applications: 1.1,1.2, 1.3, 1.4, 1.5, 2.1, 2.2, 2.3, 2.4, 2.5*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 3 Test	x	x	x	x
Chapter 3 Assignment	x	x	x	x

## Chapter 4 –Curve Sketching

Time: 11 hours

### Description

In this chapter, student use the second derivative to identify the interval of concavity and use the second derivative test to determine if a function has a local maximum or minimum at the critical points. In the end of the unit, students are able to use the curve sketching algorithm ( x-intercepts, y-intercepts, critical points, interval of increase and decrease, point of infection, and interval of concavity) to sketch a proper graph of a polynomial and rational functions.

### Specific Expectations

A. *Derivatives and their Applications:* 1.1,1.2,1.3,1.4,1.5,2.1,2.2,2.3,2.4,2.5

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 4 Test	x	x	x	x

## Chapter 5 – Derivatives of Exponential and Trigonometric Functions

Time: 14 hours

### Description

This chapter starts with an introduction to the Euler's number ( $e$ ). Students determine the derivative of Euler's number and other exponential functions. They use the concepts used in other units like, product rule, quotient rule, chain rule and curve sketching for the exponential functions. Students are also determining the derivative of sinusoidal functions and they solve related problems involving trigonometric functions.

### Specific Expectations

*A. Rate of change: 2.4,2.5,2.6,2.7,2.8*

*B. Derivatives and their Applications: 2.3,2.4*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 5 Test	x	x	x	x
Chapter 5 Assignment	x	x	x	x



## Chapter 6 – An Introduction to Vectors

Time: 14 hours

### Description

In this chapter, students will tell the difference between a scalar and vector quantity. They will represent vectors as directed line segments and perform the operations of addition, subtraction, and scalar multiplication on geometric vectors. Cartesian vectors are represented in two-space and three-space as ordered pairs and triples, respectively. The addition, subtraction, and scalar multiplication of Cartesian vectors are all investigated in this unit. Students investigate the concepts of linear dependence and independence, and collinearity and coplanarity of vectors.

### Specific Expectations

*A. Geometry and Algebra of Vectors: 1.1,1.2,1.3,1.4, 2.1,2.2*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 6 Test	x	x	x	x

## Chapter 7 – Application of Vectors

Time: 14 hours

### Description

Applications involving work and torque are used to introduce and lend context to the dot and cross products of Cartesian vectors. The vector and scalar projections of Cartesian vectors are written in terms of the dot product. The properties of vector products are investigated and proven.

### Specific Expectations

*A. Geometry and Algebra of Vectors: 2.3,2.4,2.5,2.6,2.7,2.8*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 7 Test	x	x	x	x
Chapter 6 & 7 Assignment	x	x	x	x

## Chapter 8 – Equations of Lines and Planes

Time: 10 hours

### Description

In this chapter, student will begin by determining the vector, parametric and symmetric equation of lines in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ . Student will go on to determine the vector, parametric, symmetric and scalar equation of planes in 3-spaces. The intersection of lines in 3-space and the intersections of lines in 3-spaces will also be learned. Student will determine the distance between a point and a line and also between a point and a plane.

### Specific Expectations

*A. Geometry and Algebra of Vectors: 3.1,3.2,3.3,4.1,4.2,4.3,4.4,4.5,4.6,4.7*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Homework questions	x	x	x	x
Practice Worksheets	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 8 Test	x	x	x	x

## Chapter 9 – Relationships between Points, Lines, and Planes

Time: 9 hours

### Description

In this chapter, students are introduced to the most important idea associated with vectors, the solution of systems of equations. Geometrically, the solution of two equations in two unknowns is the point of intersection between two lines on the  $xy$ -plane. In this chapter, the students are going to extend these ideas and consider systems of equations in  $\mathbf{R}^3$  and interpret their meaning.

### Specific Expectations

*A. Geometry and Algebra of Vectors: 3.1,3.2,3.3,4.1,4.2,4.3,4.4,4.5,4.6,4.7*

<b>Assessment For Learning (AFL)</b>	<b>K/U</b>	<b>T</b>	<b>A</b>	<b>C</b>
Video Lessons	x	x	x	x
Note taking	x	x	x	x
<b>Assessment As Learning (AAL)</b>				
Socratic questioning	x			
Math journal – study log				x
Class Discussion	x			x
<b>Assessment Of Learning (AOL)</b>				
Chapter 9 ISU Assignment	x	x	x	x

## Teaching/Learning Strategies

A variety of strategies are used to allow students many opportunities to attain the necessary skills for success in this course and at university. The teacher uses a variety of whole class, small group and individual activities to facilitate learning. The following is a list of specific teaching/learning strategies that the teacher may use but is not limited to:

- Lecture
- Modeling/Direct Instruction
- Demonstration/exemplars
- Videos
- Graphic organizers (Venn Diagram, T-charts, KWL charts, Placemats)
- Problem-Solving
- Homework questions
- Structured Discussions
- Student Presentation
- Jigsaw
- Brainstorming
- Self-Assessment
- Peer-Assessment
- Teacher feedback
- Group work
- Pair work
- Independent work
- Exit/Entrance Cards
- Tests
- Quizzes
- Exam

### ONLINE & OFFLINE COMPONENTS

The design of this course is intended to offer a rich balance between online and offline elements.

The following is a summary of the course components and their delivery format. Please refer to the individual unit outlines for specific details. Course content & instruction: *online* Communication between teacher and students: *online & offline* Collaboration between students: *online* Assessment & evaluation: *online & offline* Practise exercises, textbook work, readings etc: *offline*

## Assessment/Evaluation Strategies

Diagnostic assessment is used at the beginning of a unit to assist in determining a starting point for instruction. Assessment for Learning (AFL) provides information to students as they are learning and refining their skills. Assessment as Learning (AAL) acts as a stepping-stone for students to begin applying their understanding using critical thinking; it bridges the gap between AFL and AOL. Assessment of Learning (AOL), at the end of units and course, provides students with the opportunity to synthesize/apply/demonstrate their learning and the achievement of the expectations. The following is a list of specific assessment/evaluation strategies that the teacher may use but is not limited to:

**Strategies actually used in the classroom are indicated in the chart above and reflected in classroom instruction:**

**Levels:** There are four levels of achievement for students who are passing the course:

- Level 1 (50-59%)
- Level 2 (60-69%)
- Level 3 (70-79%)
- Level 4 (80-100%)

**Level 3 is the provincial standard for student achievement.**

**Final Grade:** The final grade will include the following weighting:

<b>Knowledge and understanding</b>	30%
<b>Thinking and inquiry</b>	20%
<b>Communication</b>	20%
<b>Application</b>	30%

The evaluation for the course is broken down as follows:

**70%      COURSE WORK**  
 This portion of the mark is based on performance on assignments and tests throughout the course. This portion of the grade will reflect the student's most consistent level of achievement throughout the course, although special consideration will be given to more recent evidence of achievement

**30%      COURSE CULMINATING ACTIVITIES**  
 All students will write a final examination, which will take into account the entire course, including the student's most recent and most consistent performance. The final evaluation may take the form of a written examination, an independent study project, a presentation, etc. or a combination of these formats.

**TOTAL:      100%**

## Program Planning

In order to accommodate students' needs, the teachers of UMC High School incorporate appropriate considerations in their program planning and delivery. These considerations may include, but not be limited to:

- ❑ Anticipate students' anxiety and be prepared to provide support and encouragement as they adapt to your expectations
- ❑ Help student activate prior knowledge in math
- ❑ Reading aloud strategies that enable ESL students to develop their oral communication skills
- ❑ Provide step-by-step instructions
- ❑ Provide opportunities to learn in a variety of ways – individually, cooperatively, independently, and collaboratively with teacher direction, through investigation involving hands-on experience, and through examples followed by practice
- ❑ Use concrete learning tools, such as connecting cubes, measurement tools, algebra tiles, and number cubes, invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways
- ❑ Use graphical and algebraic representations to represent mathematical problems
- ❑ Link mathematical concepts learned in class with real-world applications
- ❑ Solicit the alternative answers or elaboration to provide material for comparison, contrast, and assessment
- ❑ Encourage students to develop a willingness to persist, to investigate, to reason, to explore alternative solutions, to view challenges as opportunities to extend their learning
- ❑ Communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions
- ❑ Develop, select, apply, compare, and adapt a variety of problem – solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding